PROBLEM SETS

Basic

The Fisher equation tells us that the real interest rate approximately equals the nominal rate minus the inflation rate. Suppose the inflation rate increases from 3% to 5%. Does the Fisher equation imply that this increase will result in a fall in the real rate of interest? Explain.

- You've just stumbled on a new dataset that enables you to compute historical rates of return on U.S. stocks all the way back to 1880. What are the advantages and disadvantages in using these data to help estimate the expected rate of return on U.S. stocks over the coming year?
- 3. You are considering two alternative 2-year investments: You can invest in a risky asset with a positive risk premium and returns in each of the 2 years that will be identically distributed and uncorrelated, or you can invest in the risky asset for only 1 year and then invest the proceeds in a risk-free asset. Which of the following statements about the first investment alternative (compared with the second) are true?
  - a. Its 2-year risk premium is the same as the second alternative.
  - b. The standard deviation of its 2-year return is the same.
  - c. Its annualized standard deviation is lower.
  - d. Its Sharpe ratio is higher.
  - e. It is relatively more attractive to investors who have lower degrees of risk aversion.
- 4. You have \$5,000 to invest for the next year and are considering three alternatives:
- Intermediate
- a. A money market fund with an average maturity of 30 days offering a current yield of 6% per
- b. A 1-year savings deposit at a bank offering an interest rate of 7.5%.
- c. A 20-year U.S. Treasury bond offering a yield to maturity of 9% per year.

What role does your forecast of future interest rates play in your decisions?

- Use Figure 5.1 in the text to analyze the effect of the following on the level of real interest rates:
- a. Businesses become more pessimistic about future demand for their products and decide to reduce their capital spending.
- b. Households are induced to save more because of increased uncertainty about their future Social Security benefits.
- c. The Federal Reserve Board undertakes open-market purchases of U.S. Treasury securities in order to increase the supply of money.
- You are considering the choice between investing \$50,000 in a conventional 1-year bank CD offering an interest rate of 5% and a 1-year "Inflation-Plus" CD offering 1.5% per year plus the rate of inflation.
  - a. Which is the safer investment?
  - b. Which offers the higher expected return?
  - c. If you expect the rate of inflation to be 3% over the next year, which is the better investment? Why?
  - d. If we observe a risk-free nominal interest rate of 5% per year and a risk-free real rate of 1.5% on inflation-indexed bonds, can we infer that the market's expected rate of inflation is 3.5% per year?

Suppose your expectations regarding the stock price are as follows:

State of the Market	Probability	Ending Price	HPR (including dividends)
Boom ,	.35	\$140	44.5%
Normal growth	.30	110	14.0
Recession	.35	80	<b>~16.5</b>

Use Equations 5.11 and 5.12 to compute the mean and standard deviation of the HPR on stocks.

you are faced with the probability distribution of the HPR on the stock market index fund given in Spreadsheet 5.1 of the text. Suppose the price of a put option on a share of the index fund with exercise price of \$110 and time to expiration of 1 year is \$12.

Challenge

- a. What is the probability distribution of the HPR on the put option?
- b. What is the probability distribution of the HPR on a portfolio consisting of one share of the index fund and a put option?
- a. In what sense does buying the put option constitute a purchase of insurance in this case?
- 17. Take as given the conditions described in the previous problem, and suppose the risk-free interest rate is 6% per year. You are contemplating investing \$107.55 in a 1-year CD and simultaneously buying a call option on the stock market index fund with an exercise price of \$110 and expiration of 1 year. What is the probability distribution of your dollar return at the end of the year?
- 18. Consider these long-term investment data:
  - The price of a 10-year \$100 par zero coupon inflation-indexed bond is \$84.49.
  - A real-estate property is expected to yield 2% per quarter (nominal) with a SD of the (effective) quarterly rate of 10%.
    - a. Compute the annual rate on the real bond.
    - b. Compute the CC annual risk premium on the real-estate investment.
    - c. Use the appropriate formula and Excel Solver or Goal Seek to find the SD of the CC annual excess return on the real-estate investment.
    - d. What is the probability of loss or shortfall after 10 years?

Given \$100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills (U.S. Treasury bills) based on the following table?

Action	Probability	Expected Return
Invest in equities —		\$50,000
	.4 ——	-\$30,000
Invest in risk-free T-bill	1.0	\$ 5,000

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2) Based on the scenarios below, what is the expected return for a portfolio with the following return profile?

/ m.		Market Condition		
	Bear	Normal	Bull	
Probability	.2	.3	.5	
Rate of return	-25%	10%	24%	

Use the following scenario analysis for Stocks X and Y to answer CFA Problems 3 through 6 (round to the nearest percent).

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.5	0.3
Stock X	-20%	18%	50%
Stock Y	-15%	20%	10%

- 3. What are the expected rates of return for Stocks X and Y?
- 4. What are the standard deviations of returns on Stocks X and Y?

Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 poets with weights as follows:

$W_{bills}$	$W_{index}$
0	1.0
0.2	0.8
0.4	0.6
0.6	0.4 0.2
8.0	0.2
1.0	0

Calculate the utility levels of each portfolio of Problem 10 for an investor with A = 2. What do you conclude?

12. Repeat Problem 11 for an investor with A = 3. What do you conclude?

**UsetheseinputsforProblems13through19:**Youmanageariskyportfoliowithexpectedrate of return of 18% and standard deviation of 28%. The T-bill rate is 8%.

- 13. Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund. What is the expected value and standard deviation of the rate of return on his portfolio?
- 14. Suppose that your risky portfolio includes the following investments in the given proportions:

Stock A	25%
Stock B	32%
Stock C	43%

What are the investment proportions of your client's overall portfolio, including the position in T-bills?

- 15. What is the reward-to-volatility ratio (S) of your risky portfolio? Your client's?
- 16. Draw the CAL of your portfolio on an expected return-standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.
- 17. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16%.
  - a. What is the proportion y?
  - b. What are your client's investment proportions in your three stocks and the T-bill fund?
  - c. What is the standard deviation of the rate of return on your client's portfolio?
- 18. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%.
  - a. What is the investment proportion, y?
  - b. What is the expected rate of return on the complete portfolio?

19) Your client's degree of risk aversion is A = 3.5.

- a. What proportion, y, of the total investment should be invested in your fund?
- b. What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?
- 20. Look at the data in Table 6.7 on the average risk premium of the S&P 500 over T-bills, and the standard deviation of that risk premium. Suppose that the S&P 500 is your risky portfolio.
  - a. If your risk-aversion coefficient is A=4 and you believe that the entire 1926-2012 period is representative of future expected performance, what fraction of your portfolio should be allocated to T-bills and what fraction to equity?
  - b. What if you believe that the 1968-1988 period is representative?
  - c. What do you conclude upon comparing your answers to (a) and (b)?

## the following data in answering CFA Problems 1–3:

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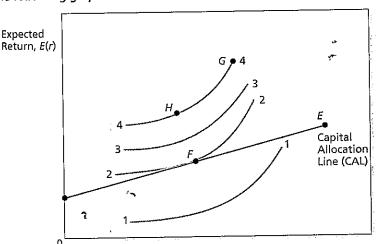
## **Utility Formula Data**

 Investment	Expected Return, E(r)	Standard Deviation, σ
	.12	.30
2	.15	.50
3	.21	.16
4	.24	.21

 $U = E(r) - \frac{1}{2}A\sigma^2$ , where A = 4

- On the basis of the utility formula above, which investment would you select if you were risk averse with A=4?
- On the basis of the utility formula above, which investment would you select if you were risk neutral?
- The variable (A) in the utility formula represents the:
- a. investor's return requirement.
- b. investor's aversion to risk.
- c. certainty equivalent rate of the portfolio.
- d. preference for one unit of return per four units of risk.

## Use the following graph to answer CFA Problems 4 and 5.



Risk, o

- Which indifference curve represents the greatest level of utility that can be achieved by the investor?
  - 5. Which point designates the optimal portfolio of risky assets?
  - 6. Given \$100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills on the basis of the following table?

Action	Probability	Expected Return
Invest in equities	.6	\$50,000
The second secon		\$30,000
Invest in risk-free T-bills	1.0	\$ 5,000